UNIT-V

ELECTRO-DYNAMIC FIELDS

Faraday's Laws of electromagnetic Induction - Induced EMF - Static and dynamic induced EMF, Maxwell's equations - modification of Maxwell's equations for time varying fields, differential and integral forms, poynting theorem and Poynting vector, Simple problems.

INTRODUCTION:

MAXWELL's EQUATIONS

Static fields

- Stationary charges → $\boxed{\text{Electrostatic fields}}$

$$\left\{ \begin{array}{l} \vec{E}\,(x,y,z) \\ \vec{E}\,(r,\phi,z) \\ \vec{E}\,(r,\theta,\phi) \end{array} \right\}$$

- Steady Currents or DC → $\boxed{\text{Magnetostatic fields}}$

$$\left\{ \begin{array}{l} \vec{H}\,(x,y,z) \\ \vec{H}\,(r,\phi,z) \\ \vec{H}\,(r,\theta,\phi) \end{array} \right\}$$

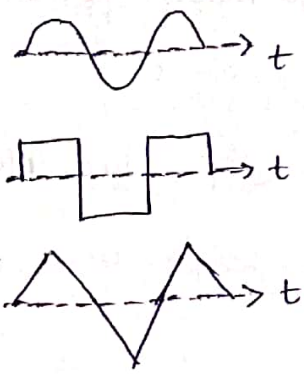
Time Varying Currents ------→ Electromagnetic Waves (or) EM Waves

$\vec{E}\,(x,y,z,t)$ & $\vec{H}\,(x,y,z,t)$

→ Sinusoidal

→ Rectangular

→ Triangular



Time Varying fields

Two major Concepts

1) Electromotive force (e.m.f)

Based on Faraday's Experiments.

2) Displacement Current

Result from Maxwell's Hyphothesis

FARADAY's LAW :

→ After Oersted's experimental discovery (up on which Biot-Savart and Ampere based their laws) that a steady Current produces a magnetic field, it seemed logical to find out wheather magnetism would produce electricity.

→ In 1831, about 11 years after Oersted's discovery, Michael Faraday in London and Joseph Henry in New York discovered that a time-varying magnetic field would produce an electric current.

→ According to Faraday's experiment, a static magnetic field produces no current flow ; but in a closed circuit, a time-varying field produces an induced voltage (called electromotive force or simply emf) that causes a flow of current.

NOTE $\vec{B}(x,y,z) \approx$ produce NO Current

$\vec{B}(x,y,z,t) \approx$ produce e.m.f in Closed circuit.

"Faraday discovered that the induced emf, V_{emf} (in volts), in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit".

This is called Faraday's law and it can be expressed as

$$\boxed{\begin{aligned} &\text{Induced emf} \;\Big\}\quad V_{emf} = -\frac{d\lambda}{dt} \quad \text{Volts} \\ &\qquad\qquad V_{emf} = \underbrace{-}_{\text{Lenz's Law}} N \cdot \frac{d\phi}{dt} \quad \text{Volts} \end{aligned}}$$

where $\lambda = N\phi$

- ϕ → Flux through each turn (wb)
- N → No. of turns in the circuit
- λ → Magnetic Flux Linkage.

→ The negative sign shows that the induced voltage acts in such a way as to oppose the flux producing it. This behaviour is described by Lenz's Law.

→ Sources of emf include electric generators, batteries, thermocouples, fuel cells, and photovoltaic cells, which all convert non electrical energy into electrical energy.

→ Consider the electric circuit of figure., where the battery is a source of emf. The electrochemical action of the battery results in an emf-produced field E_f. Due to the accumulation of charge at the battery terminals, an electro-static field $E_e (= -\nabla V)$ also exist.

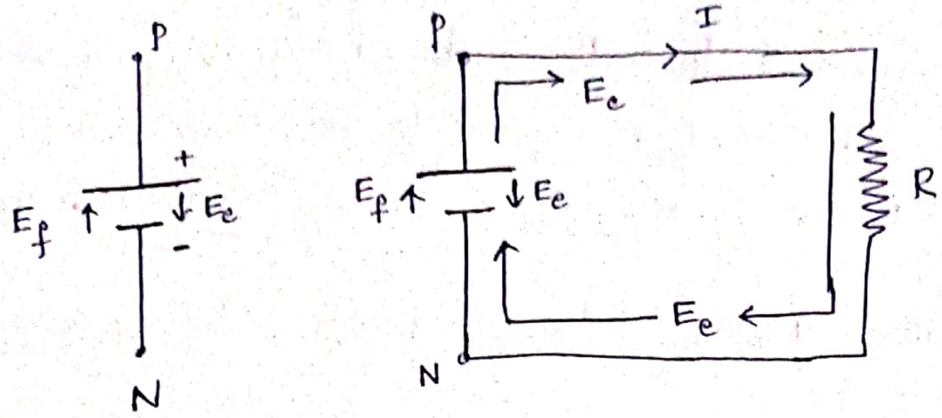


Fig: A circuit showing emf-producing field, E_f and electrostatic field, E_e.

The total electric field at any point is

$$\boxed{\vec{E} = \vec{E_f} + \vec{E_e}}$$

$\because \vec{E_f} = -\vec{E_e}$

or $\vec{E_f} + \vec{E_e} = \vec{E}$

Note: ⊛ $\vec{E_f}$ is zero outside the battery

⊛ $\vec{E_f}$ & $\vec{E_e}$ have opposite directions in the battery, and the direction of $\vec{E_e}$ inside the battery is opposite to that outside it.

Now integrate over the closed circuit

$$\oint_L \vec{E} \cdot \vec{dl} = \oint_L \vec{E_f} \cdot \vec{dl} + \oint_L \vec{E_e} \cdot \vec{dl}$$

0

where $\oint \vec{E_e} \cdot \vec{dl} = 0$ because $\vec{E_e}$ is conservative.

⊛, The emf of the battery is the line integral of the emf-produced field, i.e.,

$$\text{So, } \oint_L \vec{E} \cdot \vec{dl} = \int_N^P \vec{E_f} \cdot \vec{dl} \text{ (through battery)}$$

$$= V_{emf}$$

Also, $\vec{E_f} = -\vec{E_e}$ (with in the battery)

So, $V_{emf} = \int_N^P \vec{E_f} \cdot \vec{dl} = -\int_N^P \vec{E_e} \cdot \vec{dl} = IR$

~~TRANSFOR~~

NOTE (1) An electrostatic field $\vec{E_e}$ Cannot maintain a steady current in a closed circuit, since

$$\oint_L \vec{E_e} \, \vec{dl} = 0 = IR$$

(2) An emf-produced field $\vec{E_f}$ is non Conservative.

(3) Except in electrostatics, Voltage and potential difference are usually not equivalent.

TRANSFORMER AND MOTIONAL ELECTROMOTIVE FORCES :

Having Considered the Connection between emf and electric field, we may examine how Faraday's law links electric and magnetic fields.

Since $\boxed{V_{emf} = -\frac{d\phi}{dt}}$ — (1) ($\because N=1$, Circuit with Single turn)

Also $\boxed{V_{emf} = -\frac{d}{dt} \int_S \vec{B} \cdot ds}$ where $\phi = \int_S \vec{B} \cdot ds$ → Magnetic flux

Interms of $\vec{B}$

$$\boxed{V_{emf} = \oint_L \vec{E} \cdot \vec{dl}}$$

Interms of $\vec{E}$

So,

$$\boxed{V_{emf} = \oint_L \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s}} \longrightarrow (2)$$

In time-varying situation, both electric & magnetic fields are present and are interrelated.

Note: The variation of flux in time may be caused in three ways

(1) By having a stationary loop in a time-varying $\vec{B}$ field.

(2) By having a time-varying loop area in a static $\vec{B}$ field.

(3) By having a time-varying loop area in a time-varying $\vec{B}$ field.

(1) Stationary loop in a time-varying $\vec{B}$ field

"TRANSFORMER EMF"

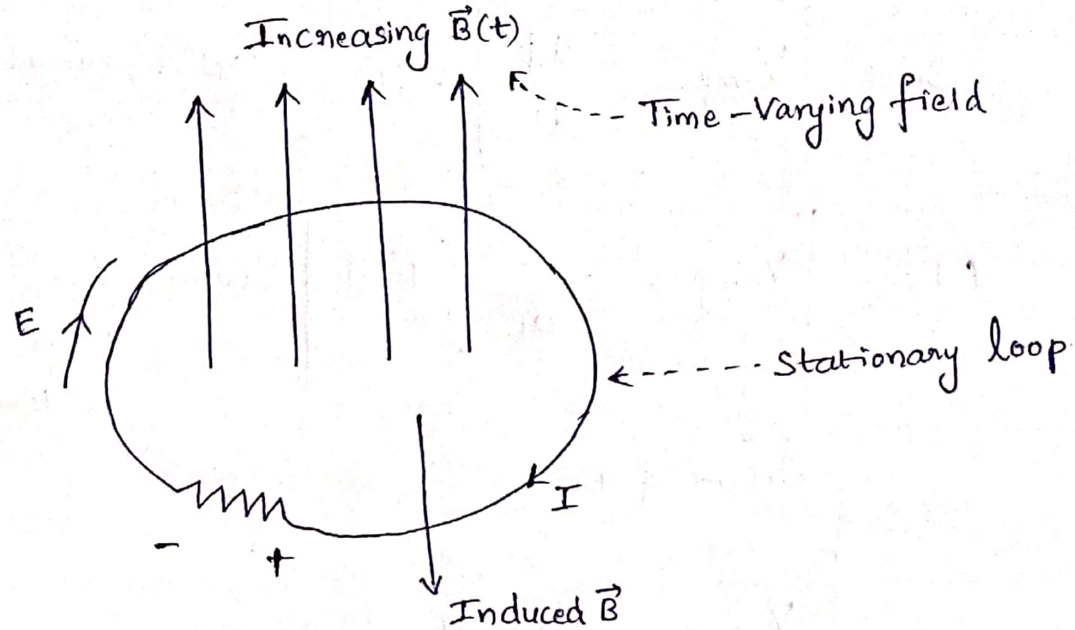


Fig: Induced emf due to stationary loop in a time-varying $\vec{B}$ field.

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We Know

$$\boxed{V_{emf} = \oint_L \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial B}{\partial t}\cdot ds}$$

→ time-varying field

$$\left[\because -\frac{d}{dt}\int_S B\cdot ds = -\int_S \frac{\partial B}{\partial t}\, ds\right]$$

Due to transformer action

→ This emf induced by the time-varying Current (producing the time-varying $\vec{B}$ field) in a stationary loop is often referred to as transformer emf in power analysis, since it is due to transformer action.

Using stoke's theorem

$$\oint_L \vec{E}\cdot d\vec{l} = \int_S (\nabla\times\vec{E})\cdot ds$$

(Closed path define open surface)

$$\Rightarrow \int_S \nabla\times\vec{E}\cdot ds = -\int_S \frac{\partial B}{\partial t}\cdot ds$$

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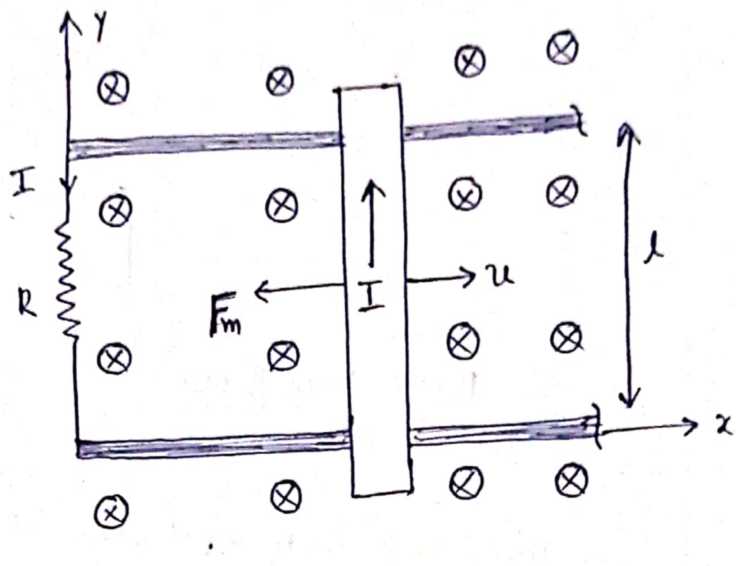
$$\boxed{\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

..... Maxwell's equation for time-varying fields.

→ It shows that the time-varying $\vec{E}$ field is not Conservative $(\nabla\times\vec{E}\neq 0)$

(2) Moving loop in a static $\vec{B}$ field

"MOTIONAL EMF"



→ When a conducting loop is moving in a static $\vec{B}$ field, an emf is induced in the loop.

→ The force on a charge moving with uniform velocity $\vec{u}$ in a magnetic field $\vec{B}$ is

$$\boxed{\vec{F}_m = Q\,\vec{u} \times \vec{B}}$$

Fig: Induced emf due to a moving loop in a static $\vec{B}$ field.

So, Motional electric field, $\vec{E}_m = \dfrac{\vec{F}_m}{Q} = \vec{u} \times \vec{B}$

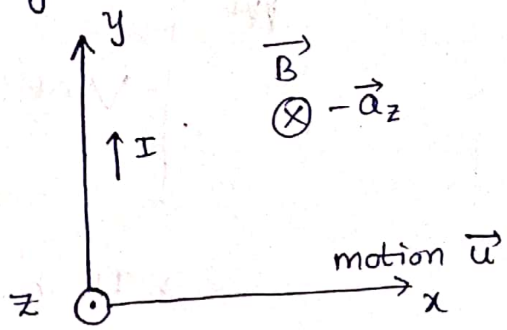
The induced emf in the loop is

$$\boxed{V_{emf} = \oint_L \vec{E}_m \cdot \vec{dl} = \oint_L (\vec{u} \times \vec{B}) \cdot \vec{dl}}$$

↓ Motional emf / Flux-cutting emf

<u>To determine Motional force:</u>

$$F_m = I\,(\underbrace{\vec{l}}_{\vec{a}_y} \times \underbrace{\vec{B}}_{-\vec{a}_z})$$



$$\vec{a}_y \times -\vec{a}_z = -\vec{a}_x$$

$$\vec{F}_m = I(\vec{l} \times \vec{B})$$
$$= \frac{Q}{t}(\vec{l} \times \vec{B})$$
$$= Q\left(\frac{\vec{l}}{t} \times \vec{B}\right)$$
$$\boxed{\vec{F}_m = Q(\vec{u} \times \vec{B})}$$

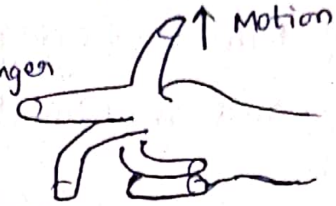
Also

$$\boxed{\vec{E}_m = \frac{\vec{F}_m}{Q} = \vec{u} \times \vec{B}}$$

→ Motional Electric field

To find I

Using Fleming's Right Hand Rule.



→ I is in +ve y-axis

The induced emf in the loop is

$$V_{emf} = \oint_L \vec{E}_m \cdot \vec{dl} = \oint_L (\vec{u} \times \vec{B}) \cdot \vec{dl}$$

Using stokes theorem

$$\oint_L \vec{E}_m \cdot \vec{dl} = \int_S (\nabla \times \vec{E}_m) \cdot \vec{ds} = \int_S \nabla \times (\vec{u} \times \vec{B}) \cdot \vec{ds}$$

(or) $$\boxed{\nabla \times \vec{E}_m = \nabla \times \vec{u} \times \vec{B}}$$

(or) $$\boxed{\vec{E}_m = \vec{u} \times \vec{B}}$$

(3) Moving loop in Time-Varying field. $\vec{B}(t)$

In the general case, a moving conducting loop is in a time-varying magnetic field. Both the transformer emf and motional emf are present.

$$\boxed{V_{emf} = \oint_L E \cdot dl = -\int_S \underbrace{\frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}}_{\text{Transformer emf}} + \oint_L \underbrace{(\vec{u} \times \vec{B}) \cdot \vec{dl}}_{\text{Motional emf}}}$$

$$\boxed{\nabla \times \vec{E} = \underbrace{-\frac{\partial \vec{B}}{\partial t}}_{\text{Transformer emf}} + \underbrace{\nabla \times (\vec{u} \times \vec{B})}_{\text{Motional emf}}}$$

DISPLACEMENT CURRENT :

Consider Maxwell's Curl equation for magnetic fields i.e., Ampere's circuit law for time-varying Conditions

$$\nabla \times \vec{H} = \vec{J} \longrightarrow (1)$$

Since, $\nabla \cdot (\nabla \times \vec{H}) = 0$

So, $\nabla \cdot \vec{J} = 0$ $\longrightarrow (2)$

In Compatible for Time-Varying Condition.

But, the Continuity of Current,

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \neq 0 \longrightarrow (3)$$

From (1)

$$\nabla \times \vec{H} = \vec{J} + \vec{J_d} \longrightarrow (4)$$

to be determined and defined.

Again,

$$\nabla \cdot (\nabla \times \vec{H}) = 0$$

$$\nabla \cdot (\vec{J} + \vec{J_d}) = 0$$

$$\nabla \cdot \vec{J} + \nabla \cdot \vec{J_d} = 0$$

or $\nabla \cdot \vec{J} = -\nabla \cdot \vec{J_d}$ or $-\nabla \cdot \vec{J} = \nabla \cdot \vec{J_d}$

Using Eq (3)

$$-\nabla \cdot \vec{J} = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot \vec{D}) = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{J_d} = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\Rightarrow \boxed{\vec{J_d} = \frac{\partial \vec{D}}{\partial t}} \dashrightarrow \text{Displacement Current density.}$$

From Eq (11)

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$ ---> Maxwell's equation for time varying fields (based on Ampere's Law)

Conduction Current density ($\vec{J} = \sigma \vec{E}$)

Note - Without the term $\vec{J_d}$, the propagation of EM waves (e.g., radio or TV waves) would be impossible.

Also, displacement Current, $I_d = \int_s \vec{J_d} \cdot ds = \int_s \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$.

GENERALIZED FORMS OF MAXWELL'S EQUATIONS :

Differential Form	Integral Form	Remarks
$\nabla \cdot \vec{D} = \rho_v$	$\oint_S \vec{D} \cdot \vec{ds} = \int_V \rho_v \, dv$	Gauss's Law
$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot \vec{ds} = 0$	Non-existence of isolated magnetic Charge / Gauss's law for magnetic field
$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$	$\oint_L \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot \vec{ds}$	Faraday's law
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\oint_L \vec{H} \cdot \vec{dl} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot \vec{ds}$	Ampere's Circuit Law

MAXWELL'S EQUATION FOR FIELDS HARMONICALLY WITH TIME:

" A time harmonic field is one that varies periodically or sinusoidally with time".

For Complex quantity

$$e^{j(\omega t+\phi)} = \cos(\omega t+\phi) + j\sin(\omega t+\phi)$$

where $j = \sqrt{-1}$

Let, $E = E_0 e^{j(\omega t+\phi)}$

$$\frac{\partial E}{\partial t} = j\omega E_0 e^{j(\omega t+\phi)} = j\omega E \quad ; \quad \boxed{\frac{\partial E}{\partial t} = j\omega E}$$

→ $D = D_0 e^{j(\omega t+\phi)}$

$$\frac{\partial D}{\partial t} = j\omega D_0 e^{j(\omega t+\phi)} = j\omega D \quad ; \quad \boxed{\frac{\partial D}{\partial t} = j\omega D}$$

→ $B = B_0 e^{j(\omega t+\phi)}$

$$\frac{\partial B}{\partial t} = j\omega B_0 e^{j(\omega t+\phi)} = j\omega B \quad ; \quad \boxed{\frac{\partial B}{\partial t} = j\omega B}$$

Differential form

$$\nabla \times \vec{E}_s = -\frac{\partial \vec{B}}{\partial t} = -j\omega \vec{B}_s$$

$$\nabla \times \vec{H}_s = \vec{J}_s + \frac{\partial \vec{D}_s}{\partial t} = \vec{J}_s + j\omega \vec{D}_s$$

$$= \sigma \vec{E}_s + j\omega \vec{D}_s = \sigma \vec{E}_s + j\omega \epsilon \vec{E}_s$$

$$= (\sigma + j\omega \epsilon)\vec{E}_s$$

$$\nabla \cdot \vec{D}_s = \rho_{v_s}$$

$$\nabla \cdot \vec{B}_s = 0$$

Integral form :

$$\oint_L \vec{E}_s \cdot \vec{dl} = -j\omega \int_S \vec{B}_s \cdot \vec{ds} \qquad \left[\because \oint_L \vec{E} \cdot \vec{dl} = \frac{-\partial}{\partial t} \int_S \vec{B} \cdot \vec{ds}\right]$$

$$\oint_L \vec{H}_s \cdot \vec{dl} = \int_S (\vec{J}_s + j\omega \vec{D}_s)\, \vec{ds} \qquad \left[\because \oint_L \vec{H} \cdot \vec{dl} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot \vec{ds}\right]$$

$$= \int_S (\sigma \vec{E}_s + j\omega\varepsilon \vec{E}_s) \cdot \vec{ds}$$

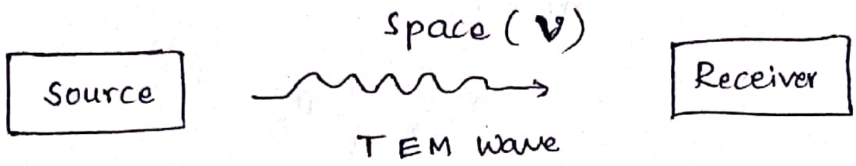
$$= (\sigma + j\omega\varepsilon) \int_S \vec{E}_s \cdot \vec{ds}$$

$$\oint \vec{D}_s \cdot \vec{ds} = \int \rho_{v_s} \, dv$$

$$\oint \vec{B}_s \cdot \vec{ds} = 0$$

WAVE POWER & THE POYNTING THEOREM

"Poynting's theorem States that the net power flowing out of a given Volume V is equal to the time rate of decrease in the energy stored with in V minus the ohmic losses."



Energy can be transported from one point (where a Transmitter is located) to another point (with a receiver) by means of EM waves.

Using Maxwell's equation.

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\text{or} \quad \nabla \times \vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\boxed{\vec{J} = \nabla \times \vec{H} - \varepsilon \frac{\partial \vec{E}}{\partial t}} \longrightarrow (1)$$

Current density

Note: $\vec{E} \cdot \vec{J} \approx$ dimensions of power

$$\vec{E} \cdot \vec{J} = \vec{E} \cdot (\nabla \cdot \vec{H}) - \varepsilon \, \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \longrightarrow (2)$$

$$\frac{1}{2} \frac{\partial}{\partial t} E^2$$

Using Vector identity,

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H})$$

$$\vec{E}\cdot\vec{J} = \vec{H}\cdot(\nabla\times\vec{E}) - \nabla\cdot(\vec{E}\times\vec{H}) - \varepsilon\left(\frac{1}{2}\frac{\partial}{\partial t}E^2\right)$$

$$\frac{-\partial\vec{B}}{\partial t} = -\mu\cdot\frac{\partial\vec{H}}{\partial t}$$

$$\vec{E}\cdot\vec{J} = -\mu\cdot\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} - \nabla\cdot(\vec{E}\times\vec{H}) - \varepsilon\left(\frac{1}{2}\frac{\partial}{\partial t}E^2\right)$$

$$\frac{1}{2}\frac{\partial}{\partial t}H^2$$

$$\text{So, } \vec{E}\cdot\vec{J} = -\mu\cdot\left(\frac{1}{2}\frac{\partial}{\partial t}H^2\right) - \nabla\cdot(\vec{E}\times\vec{H}) - \varepsilon\left(\frac{1}{2}\frac{\partial}{\partial t}E^2\right)$$

Integrate over the volume

$$\int_V \vec{E}\cdot\vec{J}\,dv = \frac{-\partial}{\partial t}\int_V\left(\frac{\mu}{2}H^2 + \frac{\varepsilon}{2}E^2\right)dv - \int_V \nabla\cdot(\vec{E}\times\vec{H})\cdot dv$$

$$\vec{J} = \sigma\vec{E}$$

divergence Theorem

$$\oint_S (\vec{E}\times\vec{H})\cdot d\vec{s}$$

$$\oint_S (\vec{E}\times\vec{H})\,d\vec{s} = -\frac{\partial}{\partial t}\int_V\left(\frac{\mu}{2}H^2 + \frac{\varepsilon}{2}E^2\right)dv - \int_V \vec{E}\cdot(\sigma\vec{E})\,dv$$

$$\boxed{\oint_S (\vec{E}\times\vec{H})\,d\vec{s} = \frac{-\partial}{\partial t}\int_V\left(\frac{\mu}{2}H^2 + \frac{\varepsilon}{2}E^2\right)dv - \int_V \sigma E^2\,dv}$$

Total power leaving the Volume = Rate of decrease in energy stored in $\vec{E}$ & $\vec{H}$ field − Ohmic Power dissipated

Poynting Theorem

$$\boxed{\vec{E} \times \vec{H} = \vec{P}}$$ ---- poynting Vector (Watts $/m^2$)

The direction of flow of power is perpendicular to $\vec{E}$ & $\vec{H}$, in the direction of $\vec{E} \times \vec{H}$ (along the direction of wave propagation $\vec{a}_K$ for uniform plane wave)

$$\boxed{\begin{aligned} \vec{P} &= \vec{E} \times \vec{H} \\ \vec{a}_K &= \vec{a}_E \times \vec{a}_H \end{aligned}}$$

So, $\vec{P}$ along $\bar{a}_K$ regarded as poynting Vector.